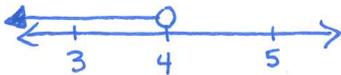


Solving Inequalities & Systems of Inequalities

Solve the following inequalities algebraically and draw a number line graph to represent each solution.

1) $8 - 3x < 4(3 - x)$

$$\begin{array}{r} 8 - 3x < 12 - 4x \\ -8 + 4x \quad -8 + 4x \\ \hline x < 4 \end{array}$$



2) $2(x + 4) - 5 \geq 5(x + 3)$

$$2x + 8 - 5 \geq 5x + 15$$

$$\begin{array}{r} 2x + 3 \geq 5x + 15 \\ -2x - 15 \quad -2x - 15 \\ \hline -12 \geq 3x \end{array}$$

$$-4 \geq x \quad (\text{SAME AS } x \leq -4)$$



3) $|5x - 16| \geq 4$

$$5x - 16 \geq 4 \quad \text{OR} \quad 5x - 16 \leq -4$$

$$\frac{5x}{5} \geq \frac{20}{5} \quad \frac{5x}{5} \leq \frac{12}{5}$$

$$x \geq 4 \quad \text{OR} \quad x \leq 2.4$$



4) $2|5x + 8| - 3 < 21$

$$\begin{array}{r} 2|5x + 8| - 3 < 21 \\ +3 \quad +3 \\ \hline 2|5x + 8| < 24 \end{array}$$

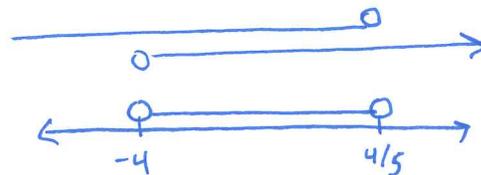
$$|5x + 8| < 12$$

$$\begin{array}{r} 5x + 8 < 12 \quad \text{and} \quad 5x + 8 > -12 \\ -8 \quad -8 \quad \quad \quad -8 \quad -8 \end{array}$$

$$\frac{5x}{5} < \frac{4}{5}$$

$$\frac{5x}{5} > \frac{-20}{5}$$

$$x < \frac{4}{5} \quad \text{and} \quad x > -4$$



5) $2x^2 - 2x - 15 \leq x^2 - 5x + 13$

$$x^2 + 3x - 28 \leq 0$$

$$x^2 + 3x - 28 = 0 \quad \boxed{-7 \leq x \leq 4}$$

$$(x + 7)(x - 4) = 0$$

$$x + 7 = 0 \quad x - 4 = 0$$

$$x = -7 \quad x = 4$$

BOUNDARY POINTS



6) $4x^2 + 3x - 5 > 3x^2 + 10x - 15$

$$x^2 - 7x + 10 > 0$$

$$x^2 - 7x + 10 = 0$$

$$(x - 5)(x - 2) = 0$$

$$x - 5 = 0 \quad x - 2 = 0$$

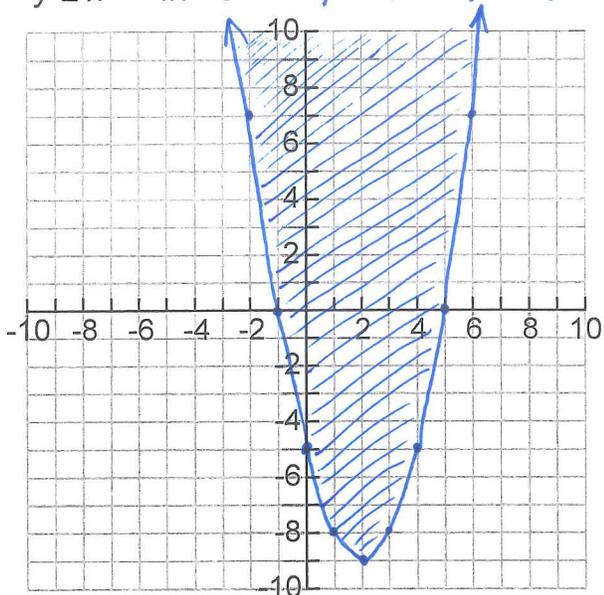
$$x = 5 \quad x = 2 \quad \leftarrow \text{BOUNDARY POINTS}$$

$$\boxed{x < 2 \quad \text{or} \quad x > 5}$$



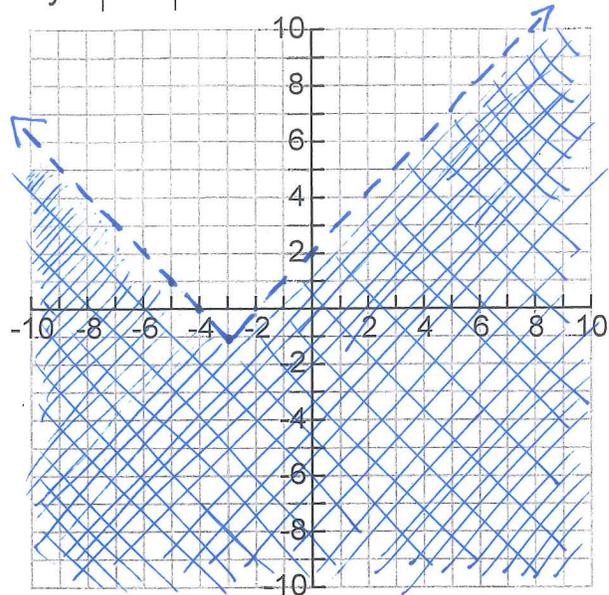
Graph the following inequalities.

7) $y \geq x^2 - 4x - 5 \rightarrow y \geq (x-2)^2 - 9$



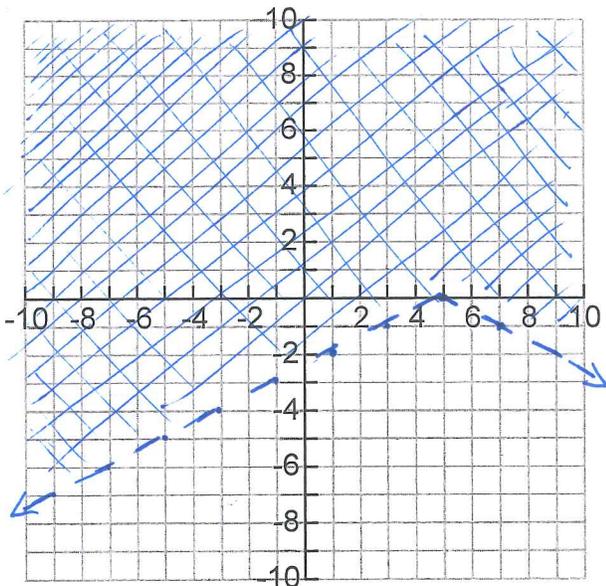
Check: $(0,0)$
 $0 \geq (0)^2 - 4(0) - 5$
 $0 \geq -5 \checkmark$

8) $y < |x+3| - 1$



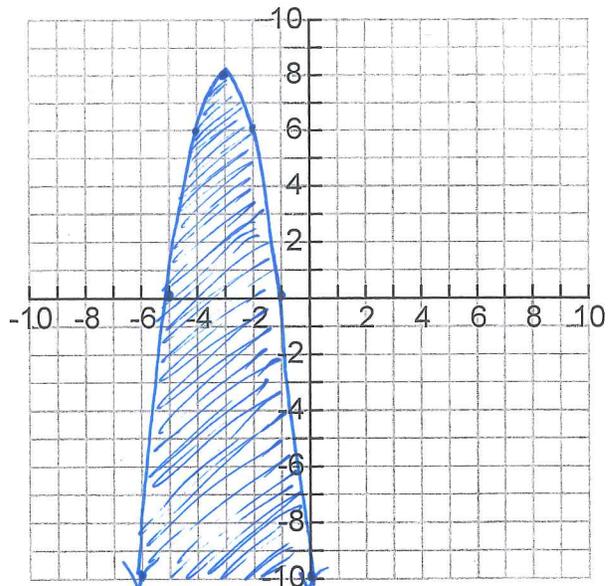
check: $(0,0)$
 $0 < |0+3| - 1$
 $0 < 2 \checkmark$

9) $y > -\frac{1}{2}|x-5|$



check: $(0,0)$
 $0 > -\frac{1}{2}|0-5|$
 $0 > -2.5 \checkmark$

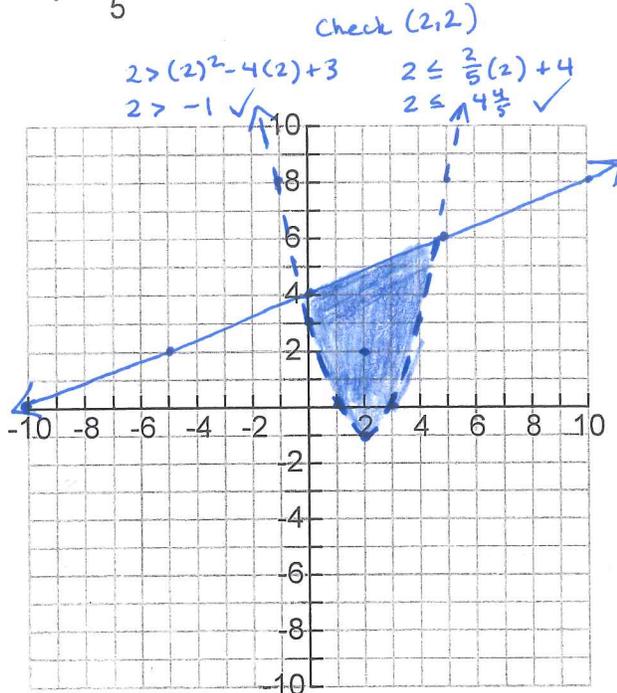
10) $y \leq -2(x+3)^2 + 8$



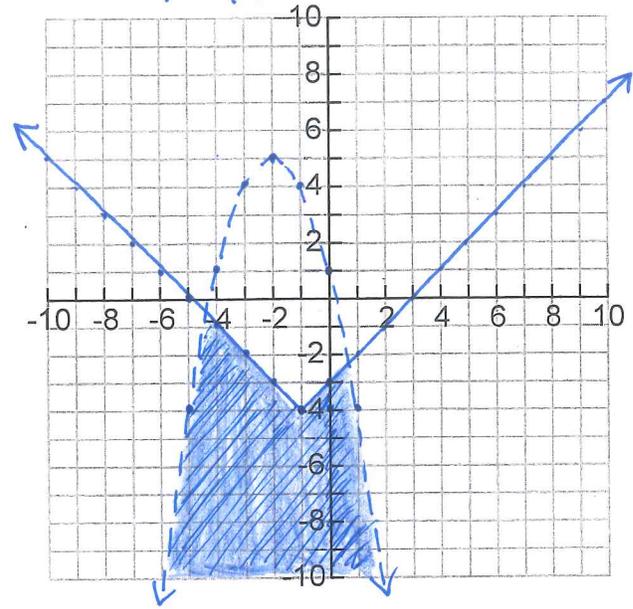
check: $(-2,-2)$
 $-2 \leq -2(-2+3)^2 + 8$
 $-2 \leq 6 \checkmark$

Graph the following systems of inequalities

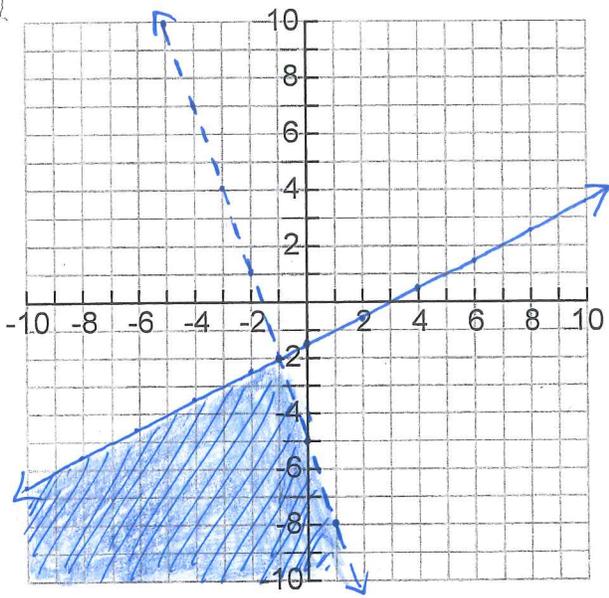
11) $y > x^2 - 4x + 3$ $y > (x-2)^2 - 1$
 $y \leq \frac{2}{5}x + 4$



12) $y \leq |x+1| - 4$
 $y < -(x+2)^2 + 5$ Check (-4, -4)



13) $3x + y < -5$ $y < -3x - 5$
 $2x - 4y \geq 6$ $y \leq \frac{1}{2}x - \frac{3}{2}$



Check: (-2, -4)

$$3(-2) + (-4) < -5$$

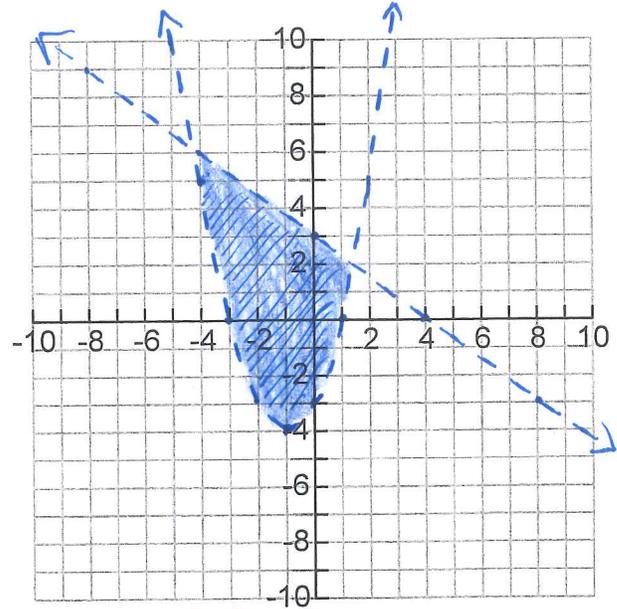
$$-10 < -5 \checkmark$$

$$2(-2) - 4(-4) \geq 6$$

$$-4 + 16 \geq 6$$

$$12 \geq 6 \checkmark$$

14) $3x + 4y < 12$ $y < -\frac{3}{4}x + 3$
 $y > (x+1)^2 - 4$



Check (0, 0)

$$3(0) + 4(0) < 12$$

$$0 < 12 \checkmark$$

$$0 > (0+1)^2 - 4$$

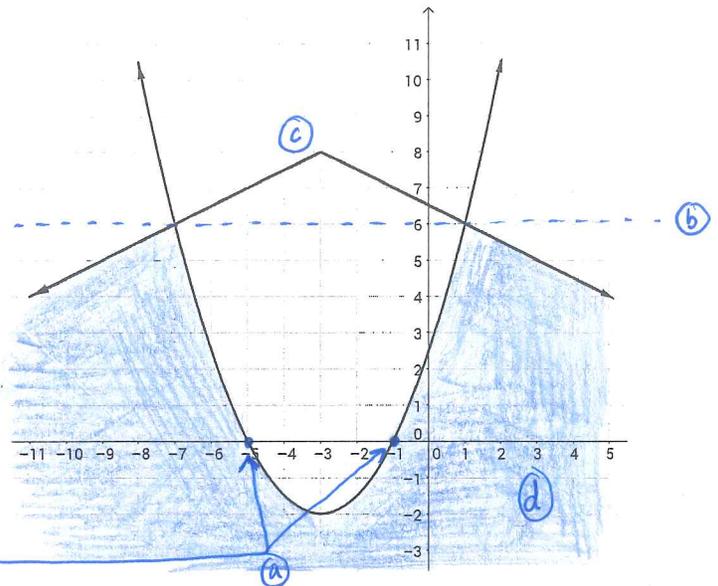
$$0 > -3 \checkmark$$

15) At right are the graphs of two functions, $f(x)$ & $g(x)$:

$$f(x) = 0.5(x+3)^2 - 2$$

$$g(x) = -0.5|x+3| + 8$$

Use the graphs to solve the following.



a) Solve for x : $0.5(x+3)^2 - 2 = 0$

$$x = -5 \text{ or } x = -1$$

b) For what values of x is $-0.5|x+3| + 8 < 6$. State your answer algebraically and graph it on a number line.

which points on the absolute value graph have outputs (y-values) that are less than "6"?

$$x < -7 \text{ or } x > 1$$



c) For what values of x is $-0.5|x+3| + 8 \geq 0.5(x+3)^2 - 2$? State your answer algebraically and graph it on a number line.

The absolute value graph is greater than the parabola when its output (y-values) are greater than the output values of the parabola for the same inputs.

i.e. parabola Abs value
i.e. $(-5, 0)$ $(-5, 7)$ ← "7" > "0"

$$-7 \leq x \leq 1$$



d) On the graph above, lightly shade the region that represents the solution to this system of inequalities.

$$f(x) \leq 0.5(x+3)^2 - 2$$

$$g(x) \leq -0.5|x+3| + 8$$

Check: $(0, 0)$

$$f(0) \leq 0.5(0+3)^2 - 2$$

$$0 \leq 0.5(0+3)^2 - 2$$

$$0 \leq 2.5 \checkmark$$

$$g(0) \leq -0.5|0+3| + 8$$

$$0 \leq -1.5 + 8$$

$$0 \leq 6.5 \checkmark$$