Algebra 2
Chapter 3 Individual Test Review

Name: _____
Date: _____ Per: _____

1. Solve the following systems algebraically. What does each solution reveal about the graph of the equations in the system?

a) $x+2y=17$
$x-y=2$

Elimination

$x+2y=17$
$2(x-y=2)2$
$2x-2y=4$

$\frac{3x}{3}=\frac{21}{3}$

$\boxed{x=7}$

$7-y=2$
$-7 \quad -7$
$-y=-5$
$\boxed{y=5}$

$\boxed{(7,5)}$

b) $4x+5y=11$
$2x+6y=16$

Elimination

$4x+5y=11$
$-2(2x+6y=16)(-2)$
$-4x-12y=-32$

$\frac{-7y}{-7}=\frac{-21}{-7}$

$\boxed{y=3}$

$4x+5(3)=11$
$4x+15=11$
$-15 \quad -15$
$\frac{4x}{4}=\frac{-4}{4}$

$\boxed{x=-1}$

$\boxed{(-1,3)}$

c) $4x-3y=-10$
$x=\frac{1}{4}y-1$

$4(\frac{1}{4}y-1)-3y=-10$
$y-4-3y=-10$
$-2y-4=-10$
$+4 \quad +4$
$\frac{-2y}{-2}=\frac{-6}{-2}$
$\boxed{y=3}$

$x=\frac{1}{4}(3)-1$
$x=\frac{3}{4}-1$
$\boxed{x=-\frac{1}{4}}$

$\boxed{(-\frac{1}{4},3)}$

d) $2x+y=-2x+5$
$3x+2y=2x+3y$

$2x+y=-2x+5$
$+2x \quad +2x$
$4x+y=5$

$3x+2y=2x+3y$
$-2x-3y \quad -2x-3y$
$x-y=0$

or

$x=y$ $\xrightarrow{\text{Substitute}}$ $4x+(x)=5$
$\frac{5x}{5}=\frac{5}{5}$
$x=1$
$y=1$

$\boxed{(1,1)}$

2. Solve each equation after first rewriting it in a simpler equivalent form.

a) $3(2x-1)+12=4x-3$

$6x-3+12=4x-3$
$6x+9=4x-3$
$-4x-9 \quad -4x-9$
$\frac{2x}{2}=\frac{-12}{2}$

$\boxed{x=-6}$

b) $7\left(\frac{3x}{7}+\frac{2}{7}=2\right)\cdot 7$

$3x+2=14$
$-2 \quad -2$
$\frac{3x}{3}=\frac{12}{3}$
$\boxed{x=4}$

multiply by 7 to get rid of fractions.

c) $4\left(\frac{3}{4}x^2=\frac{5}{4}x+\frac{1}{2}\right)4$

multiply by 4 to get rid of fractions.

$3x^2=5x+2$
$3x^2-5x-2=0$
$(3x+1)(x-2)=0$
$3x+1=0 \quad x-2=0$
$\boxed{x=\frac{-1}{3} \text{ or } x=2}$

d) $4x(x-2)=(2x+1)(2x-3)$

$4x^2-8x=4x^2-6x+2x-3$
$4x^2-8x=4x^2-4x-3$
$-4x^2 \quad -4x^2$
$-8x=-4x-3$
$+4x \quad +4x$
$\frac{-4x}{-4}=\frac{-3}{-4}$

$\boxed{x=\frac{+3}{4}}$

3. Which of the following pairs of equations or expressions are equivalent? Justify your reasoning either by using algebra to transform the first equation or expression into the second or by demonstrating with a counterexample.

a) $(2x-1)^2$; $4x^2-1$

$4x^2-4x+1 \qquad 4x^2-1$

$4(3)^2-4(3)+1 \qquad 4(3)^2-1$

$36-12+1 \qquad 4\cdot 9-1$

$25 \neq 35$

$\boxed{\text{NOT EQUIVALENT}}$

SAME INPUT GIVES DIFFENT OUTPUT.

b) $\left(\dfrac{4x^{12}}{-2x^8}\right)^3$; $-8x^{12}$

$\boxed{\text{EQUIVALENT}}$ * AS LONG AS $x \neq 0$.

$(-2x^4)^3 = \boxed{-8x^{12}}$

c) $2x-3y=6$; $y=\frac{2}{3}x+6$

$2x-3y=6$

$-2x \qquad -2x$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$

$y = \frac{2}{3}x - 2 \neq y = \frac{2}{3}x+6$

$\boxed{\text{NOT EQUIVALENT}}$

d) $\sqrt{108}$; $6\sqrt{3}$

$\sqrt{108} =$

$\sqrt{36\cdot 3}$

$\sqrt{36}\cdot\sqrt{3}$

$6\sqrt{3} = 6\sqrt{3}$

$\boxed{\text{EQUIVALENT}}$

4. Perform the indicated operation on each of the following rational expressions. Be sure to state any excluded values for the variable and that your final answer is simplified. If a graphing tool is available, check the graph of the original problem to see if it coincides with the graph of your answer.

a) $\dfrac{x^2-x-6}{x^2-9}\cdot\dfrac{x^2+5x+6}{x^2+4x+4}$

$\dfrac{\cancel{(x-3)}\cancel{(x+2)}}{\cancel{(x-3)}\cancel{(x+3)}}\cdot\dfrac{\cancel{(x+3)}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x+2)}}$

$= \boxed{1}$

$x \neq -3, 3, -2$

b) $\dfrac{\frac{x^2-1}{x}}{\frac{x^2-2x+1}{2x^2+x}}$

$\dfrac{x^2-1}{x} \div \dfrac{x^2-2x+1}{2x^2+x}$ Flipped and factored

$\dfrac{(x+1)\cancel{(x-1)}}{\cancel{x}}\cdot\dfrac{\cancel{x}(2x+1)}{\cancel{(x-1)}(x-1)}$

$\boxed{\dfrac{(x+1)(2x+1)}{x-1}}$ $x \neq 0, 1, -\frac{1}{2}$

5. Simplify the following rational expressions. For what value(s) of x is each expression undefined.

a) $\dfrac{3x(x+6)}{(x+6)(x^2-8x+12)}$

$\dfrac{3x\cancel{(x+6)}}{\cancel{(x+6)}(x-6)(x-2)}$

$\boxed{\dfrac{3x}{(x-6)(x-2)}}$

$x+6 \neq 0$ $x \neq -6$

$x-6 \neq 0$ $x \neq 6$

$x-2 \neq 0$ $x \neq 2$

$(x \neq -6, 2, 6)$

b) $\dfrac{x^2+2x-3}{x^2-2x-15}$

$\dfrac{\cancel{(x+3)}(x-1)}{\cancel{(x+3)}(x-5)}$

$\boxed{\dfrac{x-1}{x-5}}$

$x+3 \neq 0$ $x \neq -3$

$x-5 \neq 0$ $x \neq 5$

$(x \neq -3, 5)$

Even functions: opposite inputs give same output
Odd functions: Opposite inputs give opposite outputs.

6. Decide whether each function below is even, odd or neither, and explain your reasoning.

a) $y = x^3 + x$

$f(2) = (2)^3 + 2 = 10$
$f(-2) = (-2)^3 + (-2) = -10$
Odd function

b) $y = x^2 + x$

$f(3) = (3)^2 + 3 = 12$
$f(-3) = (-3)^2 + (-3) = 6$
Neither

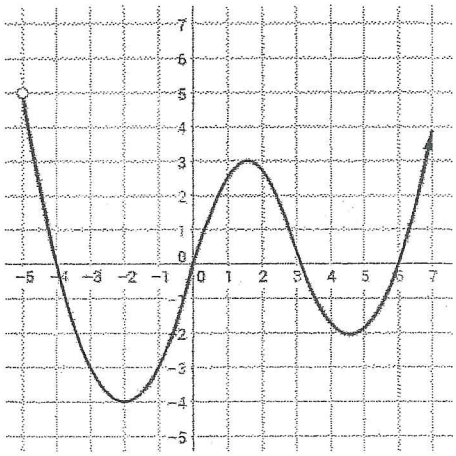
c) $y = x^4 + x^2$

$f(2) = (2)^4 + (2)^2 = 20$
$f(-2) = (-2)^4 + (-2)^2 = 20$
Even function

7. First, identify the parent graphs of the following equations. Then, describe how their graphs would be transformed from the parent graphs.

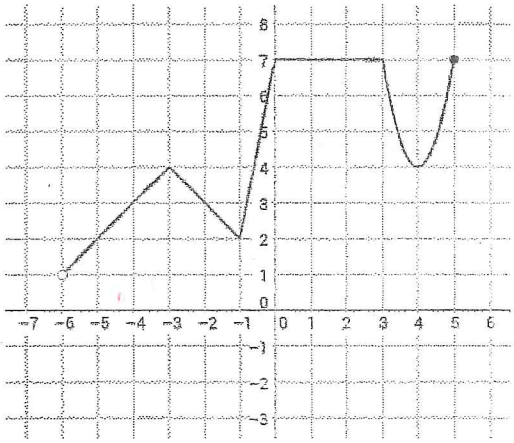
	Parent Graph	How is the graph transformed?
a) $y = 0.25(x-8)^3 + 2$	$y = x^3$	SHIFTED RIGHT 8 AND UP 2 vertically compressed by 0.25.
b) $(x+3)^2 + y^2 = 25$	$x^2 + y^2 = r^2$	Center shifted left 3 AND HAS RADIUS LENGTH 5.
c) $y = \|x-5\| + 3$	$y = \|x\|$	SHIFTED RIGHT 5 AND UP 3. SAME SHAPE AS PARNT.

8. State the domain and range for the function based on its graph.



Domain: $(-5, \infty)$

Range: $[-4, \infty)$



Domain: $(-6, 5]$

Range: $(1, 7]$

9. Find the x- and y-intercepts of $y = x^2 - 3x - 3$.

x-int: $0 = x^2 - 3x - 3$

$a=1 \quad b=-3 \quad c=-3$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-3)}}{2(1)}$

$x = \frac{3 \pm \sqrt{21}}{2}$

$x = \frac{3+\sqrt{21}}{2} \approx 3.79$

$x = \frac{3-\sqrt{21}}{2} \approx -0.79$

x-int: $(3.79, 0)$
$(-0.79, 0)$
y-int: $(0, -3)$

10. Find the center and radius of the circle by completing the square twice.

$$x^2 + y^2 + 6x - 2y - 54 = 0$$
$$+54 \quad +54$$

$x^2 + 6x + y^2 - 2y = 54$

$(x^2 + 6x + \underline{9}) + (y^2 - 2y + \underline{1}) = 54 + \underline{9} + \underline{1}$

$(x+3)^2 + (y-1)^2 = 64$

Center: $(-3, 1)$

radius 8

11. Find the slope and the distance between the points (-2, -3) and (-4, 4).

Slope = $\frac{7}{-2}$ Distance between points = $\sqrt{53}$

$m = \frac{4-(-3)}{-4-(-2)} = \frac{7}{-2}$

$d = \sqrt{(4+3)^2 + (-4+2)^2}$

$\sqrt{(7)^2 + (-2)^2}$

$\sqrt{49 + 4} = \sqrt{53}$

12. Write the equation of the parabola in graphing form. Find the vertex, x-intercepts, y-intercepts, and the equation for the line of symmetry.

$y = x^2 - 6x + 11$
$-2 \quad -2$

$y - 2 = x^2 - 6x + 9$

$y - 2 = (x-3)^2$
$+2 \quad +2$

$\boxed{y = (x-3)^2 + 2}$

Parabola opens upward and vertex is above x-axis. →

vertex $(3, 2)$

x-intercept(s) NONE

y-intercept(s) $(0, 11)$

EQ of line of symmetry $x = 3$

13. Simplify each so that there are no negative or fractional exponents in the expression.

a) $\left(v^2 g^{\frac{3}{4}}\right)^8 = v^{16} g^6$

b) $n^3(n^2)^5 = n^{13}$ $\quad n^3 \cdot n^{10} = n^{13}$

c) $(x^3 y^6)^{\frac{1}{2}} = xy^3\sqrt{x}$

$\sqrt{x^3 y^6} = \sqrt{x^3} \cdot \sqrt{y^6}$

$\sqrt{x^2 \cdot x} \cdot y^3 = \sqrt{x^2} \cdot \sqrt{x} \cdot y^3$

$x \cdot \sqrt{x} \cdot y^3$

d) $z^{-\frac{3}{4}} = \frac{1}{\sqrt[4]{z^3}}$

$\sqrt[4]{\frac{1}{z^3}}$ or $\left(\sqrt[4]{\frac{1}{z}}\right)^3$