Honors Algebra 2
Chapter 10 Individual Test Review

Name: Key
Date: ____________ Per: ___

Be prepared to answer these questions without your calculator.

1. How many ways can 2 people be chosen from 8? Write your answer in $_nP_r$ or $_nC_r$ form and find the value.

$$\boxed{_8C_2} = \frac{8\cdot 7}{2\cdot 1} = \frac{56}{2} = 28$$

2. How many ways can first, second and third placed be chosen from 6 participants? Write your answer in $_nP_r$ or $_nC_r$ form and find the value.

$$_6P_3 = 6\cdot 5\cdot 4 = 120$$

3. Write an infinite geometric series that has a finite sum. Explain how you know you can calculate the sum.

$$100 + 50 + 25 + 12.5 + 6.25 + \dots$$

The reason I know that I can calculate a sum is because the multiplier has an absolute value between 0 & 1. ($r = \frac{1}{2}$)

4. Use Pascal's Triangle to expand the binomials.

a. $(x-2)^4 = (a+b)^4$

$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(x)^4 + 4(x)^3(-2) + 6(x)^2(-2)^2 + 4(x)(-2)^3 + (-2)^4$

$x^4 - 8x^3 + 24x^2 - 32x + 16$

b. $(x+3y)^3 = (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(x)^3 + 3(x)^2(3y) + 3(x)(3y)^2 + (3y)^3$

$x^3 + 9x^2y + 27xy^2 + 27y^3$

5. Find the number of terms in each series.

a. 7 + 5 + 3 +...+ (-15)

$t(n) = 7 - 2(n-1)$

$-15 = 7 - 2n + 2$

$-15 = 9 - 2n$

$-15 = 9 - 2n$

$\frac{-24}{-2} = \frac{-2n}{-2}$

$12 = n$

b. 1 + 5 + 9 + ...+ 29

$t(n) = 1 + 4(n-1)$

$29 = 1 + 4(n-1)$

$29 = 1 + 4n - 4$

$29 = 4n - 3$

$\frac{32}{4} = \frac{4n}{4}$

$8 = n$

6. Write each series from question 5 in summation notation.

a) $\sum_{n=1}^{12} [7 - 2(n-1)]$ or $[9 - 2n]$

b) $\sum_{n=1}^{8} [1 + 4(n-1)]$ or $[4n - 3]$

7. A pendulum swings 8 inches on its first back-and-forth motion and only travels $\frac{1}{4}$ the distance on each future back-and-forth swing. How far does the pendulum travel if it swings forever (or before it stops in real life)?

$$S_{inf} = \frac{8}{1 - \frac{1}{4}} = \frac{8}{\frac{3}{4}} = \frac{8}{1}\cdot\frac{4}{3} = \frac{32}{3} \text{ inches}$$

$\boxed{10\frac{2}{3} \text{ inches}}$

You may use your calculator for these problems.

Combination and Permutation

8. Sarah needs to choose 9 people from her class of 20 for her softball team. How many possible ways can Sarah choose her teammates? Write your answer in ${}_nP_r$ or ${}_nC_r$ form and find the value.

$${}_{20}C_9 = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 167{,}960 \text{ ways}$$

9. Ms. Carr, Sarah's gym teacher, does not think it's fair for her to pick all 9 people for her team so Ms. Carr has chosen 2 people who must be on Sarah's team. How many possible ways can Sarah pick her team now if she must choose Ms. Carr's two students? Write your answer in ${}_nP_r$ or ${}_nC_r$ form and find the value.

She must now pick 7 individuals out of the 18 who are left.

$${}_{18}C_7 = \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 31{,}824 \text{ ways}$$

Arithmetic Series and Sums

10. Find the number of terms in each series.

a. 19 + 26 + 33 +... + 278

$278 = 19 + 7(n-1)$

$278 = 19 + 7n - 7$

$278 = 7n + 12$

$266 = 7n$

$266 = 7n$

$38 = n$

b. 20 + 17 + 14 +...+ -22

$-22 = 20 - 3(n-1)$

$-22 = 20 - 3n + 3$

$-22 = -3n + 23$

$-45 = -3n$

$\frac{-45}{-3} = \frac{-3n}{-3}$

$15 = n$

11. Find the **sum** for each series in problem 10 then write in summation notation.

a.

$$\sum_{n=1}^{38} [19 + 7(n-1)] \text{ or } [7n+12]$$

Sum = 5643

b.

$$\sum_{n=1}^{15} [20 - 3(n-1)] \text{ or } (-3n+23)$$

Sum = -15

12. Find the sum of the series below.

$$\sum_{n=1}^{13} 4 + 8(n-1)$$

$t(1) = 4 + 8(1-1)$

$t(1) = 4$

$t(13) = 4 + 8(13-1)$

$4 + 8(12)$

$4 + 96$

$t(13) = 100$

$S(13) = (4+100)\left(\frac{13}{2}\right)$

$(104)(6.5)$

$\boxed{S(13) = 676}$

$$\frac{\log(177147)}{\log 3} = \frac{(n)\log 3}{\log 3}$$

Geometric Series and Sums

13. Find the number of terms in each series.

a. $a_1 = 8$, $r = 3$, $S_n = 708{,}584$ $n=11$

$708{,}584 = \frac{8(3)^n - 8}{3-1}$

$708{,}584 = \frac{8(3)^n - 8}{2}$

$1417168 = 8(3)^n - 8$

$1417168 = 8(3)^n - 8$

$+8 \quad +8$

$\frac{1417176}{8} = \frac{8(3)^n}{8}$

$177147 = 3^n$

b. $-5 + -10 + -20 + \ldots + -2560$

$\frac{-2560}{-5} = \frac{(-5)(2)^{n-1}}{-5}$

$512 = 2^{n-1}$

$\frac{\log(512)}{\log 2} = \frac{(n-1)\log 2}{\log 2}$

$\frac{\log(512)}{\log(2)} = n-1$

$9 = n-1$

$10 = n$

13. Find the **sum** of each series below.

a. $4 + 20 + 100 + \ldots + 48828124$

$r=5$

$48828124(5) = 244140620$

(LAST TERM × MULTIPLIER = NEXT TERM)

$\text{Sum} = \frac{244140620 - 4}{5-1} = \boxed{61035154}$

b. $a_1 = 1.6$, $r = 2$, ~~$S_n = 4877720$~~ $n=14$

$S(14) = \frac{1.6(2)^{14} - 1.6}{2-1} = \boxed{26212.8}$

14. Write the following series in summation notation.

$6 + 12 + 24 + \ldots + 1572864$

$$\boxed{\sum_{n=1}^{19} 6(2)^{n-1}}$$

$\frac{1572864}{6} = \frac{6(2)^{n-1}}{6}$

$262144 = 2^{n-1}$

$\frac{\log(262144)}{\log(2)} = n-1$

$18 = n-1$

$\boxed{19 = n}$

Logarithmic Equations

15. Rewrite each logarithmic expression as one logarithm.

a. $\log(4x) + 3\log(2x)$

$\log(4x) + \log(2x)^3$

$\log(4x \cdot 8x^3)$

$\boxed{\log(32x^4)}$

b. $2\log(7xy) - \log(y^5)$

$\log(7xy)^2 - \log y^5$

$\log\left(\frac{49x^2y^2}{y^5}\right) = \boxed{\log\left(\frac{49x^2}{y^3}\right)}$

c. $log_2(5x) + log_2(6xy) - 2log_2(4y)$

$\log_2(30x^2y) - \log_2(16y^2)$

$\log_2 \frac{30x^2y}{16y^2} = \boxed{\log_2\left(\frac{15x^2}{8y}\right)}$

16. Solve the logarithmic equations for x.

a. $log_{16}(x) + log_{16}(x+3) = \frac{1}{2}$

$\log_{16}(x^2+3x) = \frac{1}{2}$

$16^{\frac{1}{2}} = x^2 + 3x$

$4 = x^2 + 3x$

$0 = x^2 + 3x - 4$

$0 = (x+4)(x-1)$

$x = -4$ $\boxed{x = 1}$

b. $log_4(16x) + log_4(x) = log_4(400)$

$\log_4(16x^2) = \log_4(400)$

$\frac{16x^2}{16} = \frac{400}{16}$

$x^2 = 25$

$\boxed{x = \pm 5}$

c. $2\log(x) - \log(4x) = 1$

$\log \frac{x^2}{4x} = 1$

$\log \frac{x}{4} = 1$

$10 = \frac{x}{4}$

$\boxed{40 = x}$