

2) Solve the system of equations.

A $2x + y - 3z = -12$

B $5x - y + z = 11$

C $x + 3y - 2z = -13$

A) $2x + y - 3z = -12$

B) $5x - y + z = 11$

D) $7x - 2z = -1$

2E) $32x + 2z = 40$

$$\frac{39x}{39} = \frac{39}{39}$$

$$x = 1$$

$$7(1) - 2z = -1$$

$$7 - 2z = -1$$

$$\begin{array}{r} -7 \\ \hline -2z = -8 \end{array}$$

$$-2z = -8$$

$$z = 4$$

3B) $15x - 3y + 3z = 33$

C) $y + 3y - 2z = -13$

E) $16x + z = 20$

$$2(1) + y - 3(4) = -12$$

$$2 + y - 12 = -12$$

$$y - 10 = -12$$

$$y = -2$$

$$(1, -2, 4)$$

3) Find an equation for the parabola in standard form that passes through the points $(-1, 10)$, $(0, 5)$, and $(2, 7)$.

$$y = ax^2 + bx + c$$

$$10 = a(-1)^2 + b(-1) + c \quad \text{Ⓐ} \quad 10 = a - b + c$$

$$5 = a(0)^2 + b(0) + c \quad \text{Ⓑ} \quad 5 = c$$

$$7 = a(2)^2 + b(2) + c \quad \text{Ⓒ} \quad 7 = 4a + 2b + c$$

$$\frac{10}{-5} = \frac{a - b + 5}{-5}$$

$$\text{Ⓓ} \quad 5 = a - b$$

$$\frac{7}{-5} = \frac{4a + 2b + 5}{-5}$$

$$\text{Ⓔ} \quad 2 = 4a + 2b$$

$$2 \cdot \text{Ⓓ} \quad 10 = 2a - 2b$$

$$\text{Ⓔ} \quad 2 = 4a + 2b$$

$$\frac{12}{6} = \frac{6a}{6}$$

$$2 = a$$

$$\frac{5}{-2} = \frac{2 - b}{-2}$$

$$\frac{3}{-1} = \frac{-b}{-1}$$

$$-3 = b$$

$$y = 2x^2 - 3x + 5$$

OR YOU CAN CHOOSE TO LOG FIRST

$$2^x = 17 \rightarrow \log_2(17) = x$$

4) Solve each equation to the nearest thousandth.

a) $2^x = 17$

$$y = \frac{\log(17)}{\log(2)}$$

$$\log(2)^y = \log(17)$$

$$y \cdot \log(2) = \frac{\log(17)}{\log(2)}$$

$$y = 4.087$$

b) $\frac{5(3^{x+1})}{5} = \frac{85}{5}$

$$3^{x+1} = 17$$

$$\log(3)^{x+1} = \log 17$$

$$\frac{(x+1) \cdot \log(3) = \log(17)}{\log(3) \quad \log(3)}$$

$$x+1 = \frac{\log(17)}{\log(3)}$$

$$x = \frac{\log(17)}{\log(3)} - 1 = 1.579$$

OR $\log_3(17) = x+1$
 $\log_3(17) - 1 = x$
 $1.579 = x$

$$x = 1.579$$

c) $\log_3(x+1) = 2$ * CHANGE FROM LOG FORM TO EXPONENTIAL FORM.

$$3^2 = x+1 \quad \log_b(m) = n \leftrightarrow b^n = m$$

$$9 = x+1$$

$$8 = x$$

5) Multiply/Divide and simplify. REMEMBER TO FACTOR FIRST!!

a) $\frac{3x^2 - 5x - 2}{2x^2 - 11x + 15} \cdot \frac{2x^2 - 5x}{3x^3 - 5x^2 - 2x} \rightarrow x(3x^2 - 5x - 2)$

$$\frac{(3x+1)(x-2)}{(2x-5)(x-3)} \cdot \frac{x(2x-5)}{x(3x+1)(x-2)}$$

$$= \frac{1}{x-3}$$

b) $\frac{x^2-1}{x^2-6x-7} \div \frac{x^3+x^2-2x}{x-7}$

$$\frac{x^2-1}{x^2-6x-7} \cdot \frac{x-7}{x^3+x^2-2x} \leftarrow x(x^2+x-2)$$

$$\frac{(x+1)(x-1)}{(x-7)(x+1)} \cdot \frac{x-7}{x(x+2)(x-1)}$$

$$\frac{1}{x(x+2)}$$

6) Add/Subtract and simplify. * NEED COMMON DENOMINATORS!

a) $\frac{x+2}{x^2-9} - \frac{1}{x+3}$

$$\frac{x+2}{(x+3)(x-3)} - \frac{1}{x+3} \cdot \frac{(x-3)}{(x-3)}$$

$$\frac{x+2}{(x+3)(x-3)} - \frac{x-3}{(x+3)(x-3)}$$

$$\frac{x+2 - (x-3)}{(x+3)(x-3)} = \frac{5}{(x+3)(x-3)}$$

b) $\frac{4}{x^2+5x+6} + \frac{2x}{x+2}$

$$\frac{4}{(x+3)(x+2)} + \frac{2x}{(x+2)} \cdot \frac{(x+3)}{(x+3)}$$

$$\frac{4}{(x+3)(x+2)} + \frac{2x(x+3)}{(x+3)(x+2)}$$

$$\frac{4 + 2x(x+3)}{(x+3)(x+2)} = \frac{4 + 2x^2 + 6x}{(x+3)(x+2)}$$

$$\frac{2x^2 + 6x + 4}{(x+3)(x+2)} = \frac{2(x^2 + 3x + 2)}{(x+3)(x+2)} = \frac{2(x+2)(x+1)}{(x+3)(x+2)}$$

7) Solve.

$$\log(x) + \log(2x) = 5$$

$$\log_{10}(2x^2) = 5$$

$$10^5 = 2x^2$$

$$\frac{100,000}{2} = \frac{2x^2}{2}$$

$$\sqrt{50,000} = \sqrt{x^2}$$

$$223.607 = x$$

If solving a log equation with multiple logs on the same side, you must use log properties to rewrite as a single log. Then change to its exponential form and solve.

8) An exponential function $y = a(b)^x$ passes through (3, 7.5) and (4, 6.25). Find the equation of the function.

$$y = ab^x \quad (3, 7.5) \quad (4, 6.25)$$

* Create a system of equations, then solve for a & b.

$$7.5 = ab^3 \quad 6.25 = ab^4$$

$$\frac{6.25 = ab^4}{7.5 = ab^3} = 7.5 = a\left(\frac{5}{6}\right)^3$$

$$7.5 = \frac{125}{216}a$$

$$12.96 = a$$

$$\frac{5}{6} = b$$

$$y = 12.96\left(\frac{5}{6}\right)^x$$

Answers

3) $y = 2x^2 - 3x + 5$

4b) $x = 1.579$

5a) $\frac{1}{x-3}$

6a) $\frac{5}{(x+3)(x-3)}$

7) $x = 223.607$

2) (1, -2, 4)

4a) $x = 4.087$

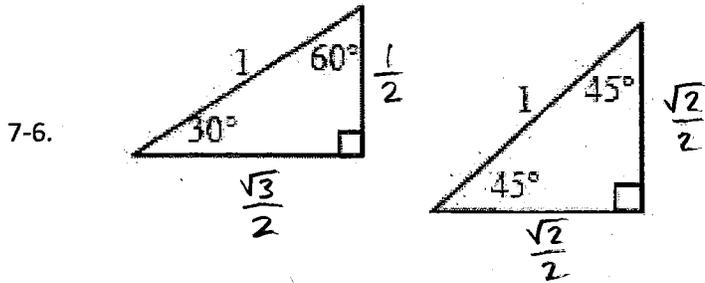
4c) $x = 8$

5b) $\frac{1}{x(x+2)}$

6b) $\frac{2(x+1)}{(x+3)}$

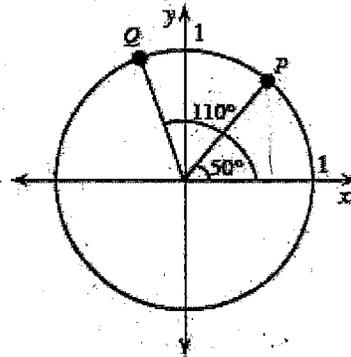
8) $y = 12.96\left(\frac{5}{6}\right)^x$

Find the missing sides of the right triangle.

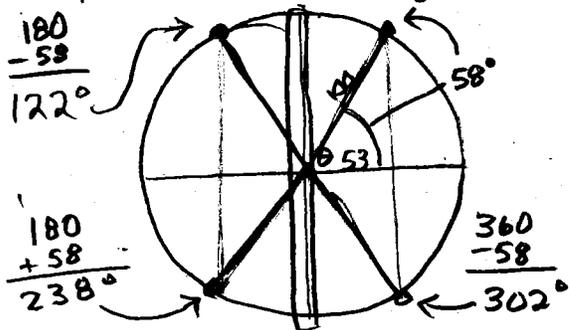


7-54. Find the coordinates of point P and Q on the unit circle.

$P(0.64, 0.77)$ $Q(-0.34, 0.94)$
 $(\cos 50, \sin 50)$ $(\cos 110, \sin 110)$



7-62. Shira was riding *The Screamer* when it broke down. Her seat was 53 horizontal feet from the central support pole. What was her seat's angle of rotation? Is there more than one possible solution?



$\cos \theta = \frac{53}{100}$
 $\theta = \cos^{-1}\left(\frac{53}{100}\right)$
 $\theta = 58^\circ, 122^\circ, 238^\circ, 302^\circ$



7-53. Use the Pythagorean Identity to find the exact coordinates of a point in the first quadrant on the unit circle that has $\sin \theta = \frac{1}{4}$.

$\cos^2 \theta + \sin^2 \theta = 1$
 $(x)^2 + \left(\frac{1}{4}\right)^2 = (1)^2$
 $x^2 + \frac{1}{16} = \frac{16}{16}$
 $\sqrt{x^2} = \sqrt{\frac{15}{16}}$
 $x = \frac{\sqrt{15}}{4}$

Point:
 $\left(\frac{\sqrt{15}}{4}, \frac{1}{4}\right)$

7-110. For an angle θ in the third quadrant, $\cos \theta = -\frac{12}{13}$. Use this information to find each of the following values without using a calculator.

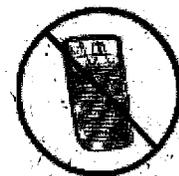
a. $\sin \theta$
 $\sin \theta = -\frac{5}{13}$

$\left(-\frac{12}{13}\right)^2 + y^2 = 1$
 $\frac{144}{169} + y^2 = \frac{169}{169}$
 $-\frac{144}{169} \quad -\frac{144}{169}$
 $\sqrt{y^2} = \sqrt{\frac{25}{169}}$

b. $\tan \theta$
 $y = \frac{5}{13}$

$\frac{\sin \theta}{\cos \theta} = \frac{-\frac{5}{13}}{-\frac{12}{13}}$

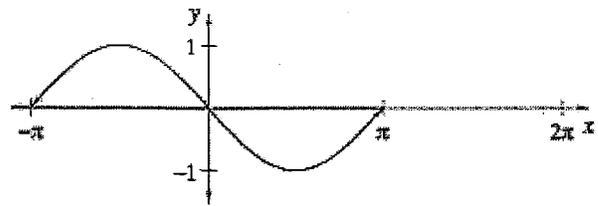
$\tan \theta = \frac{5}{12}$



7-117. The graph at the right was made by shifting the first cycle of $y = \sin x$ to the left.

a. How many units to the left was it shifted?

The graph was shifted π units to the left.

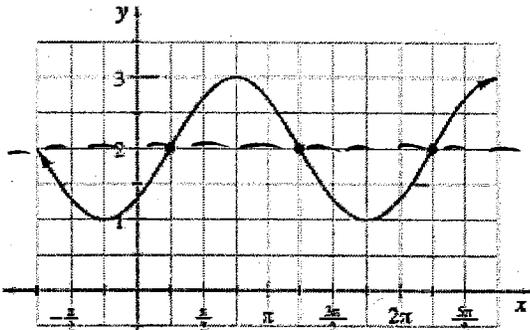


b. Transform $y = \sin x$ to represent the graph in part(a).

The equation would be: $y = \sin(x + \pi)$

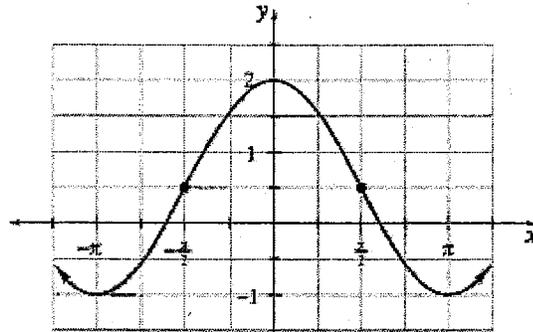
7-129. Write an equation for each graph.

a.



$$y = \sin\left(x - \frac{\pi}{4}\right) + 2$$

b.



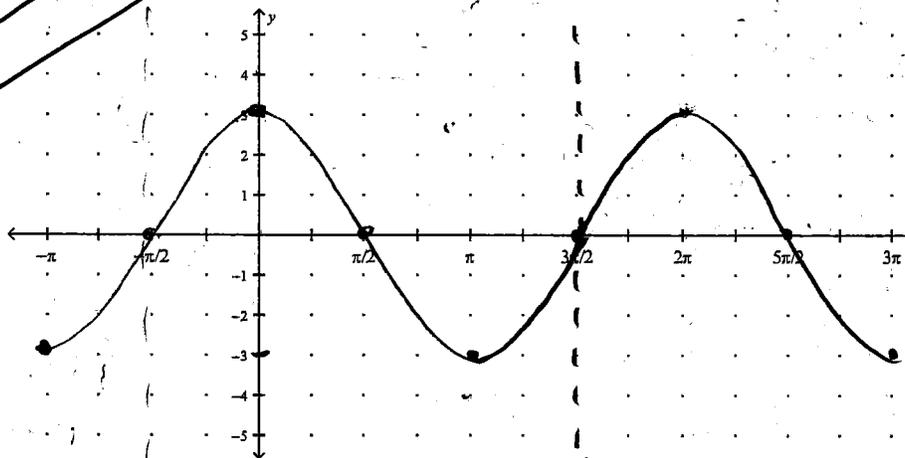
$$y = 1.5 \cos(x) + 0.5$$

OR

$$y = 1.5 \sin\left(x + \frac{\pi}{2}\right) + 0.5$$

8. Graph the equation $y = 3\sin\left(x - \frac{3\pi}{2}\right)$ from $-\pi$ to 3π .

Amplitude
Phase shift



7-40. Find the x and y intercepts of the quadratic function $y = 3x^2 + 6x + 1$.

x-int: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{-6 \pm \sqrt{24}}{6} = \frac{-6 \pm 2\sqrt{6}}{6} = \frac{-3 \pm \sqrt{6}}{3}$$

y-intercept: $y = 3(0)^2 + 6(0) + 1$

$y = 1$
 $(0, 1)$

$\left(\frac{-3 + \sqrt{6}}{3}, 0\right)$
 $\left(\frac{-3 - \sqrt{6}}{3}, 0\right)$

10. Write the equation $y = x^2 + 4x - 17$ in graphing form. Identify the vertex and y intercept.

* Completing the Square !!

$$y = x^2 + 4x - 17$$

$$\begin{array}{r} +17 \\ \hline y+17 = x^2 + 4x \end{array}$$

$$y+17 + \underline{4} = x^2 + 4x + \underline{4}$$

$$y+21 = x^2 + 4x + 4$$

$$(x+2)(x+2)$$

$$y+21 = (x+2)^2$$

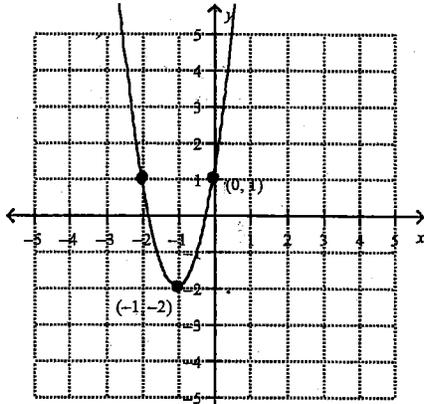
$$y = (x+2)^2 - 21$$

vertex:

$$(-2, -21)$$

$$y\text{-int: } (0, -17)$$

11. Use the graph to help write its equation in graphing form. Estimate the x-intercepts.



Estimates for x-int may vary.

$$y = a(x-h)^2 + k \quad (h, k) \text{ is vertex}$$

$$(-1, -2)$$

$$y = a(x+1)^2 - 2$$

$$1 = a(0+1)^2 - 2$$

$$1 = a - 2$$

$$\boxed{3 = a}$$

Plug in vertex for (h, k) , then a random point from parabola to solve for "a".

$$y = 3(x+1)^2 - 2$$

7-82. Write the equation in graphing form. Identify the vertex and axis of symmetry.

Completing the square w/ "a"

$$y = 3x^2 - 18x + 26$$

$$\begin{array}{r} -26 \quad -26 \\ \hline y-26 = 3x^2 - 18x \end{array}$$

$$y-26 = 3(x^2 - 6x)$$

$$y-26 = 3(x^2 - 6x + 9)$$

$$\underline{27} + y - 26 = 3(x^2 - 6x + 9)$$

$$y + 1 = 3(x^2 - 6x + 9)$$

$$y+1 = 3(x-3)^2$$

$$y = 3(x-3)^2 - 1$$

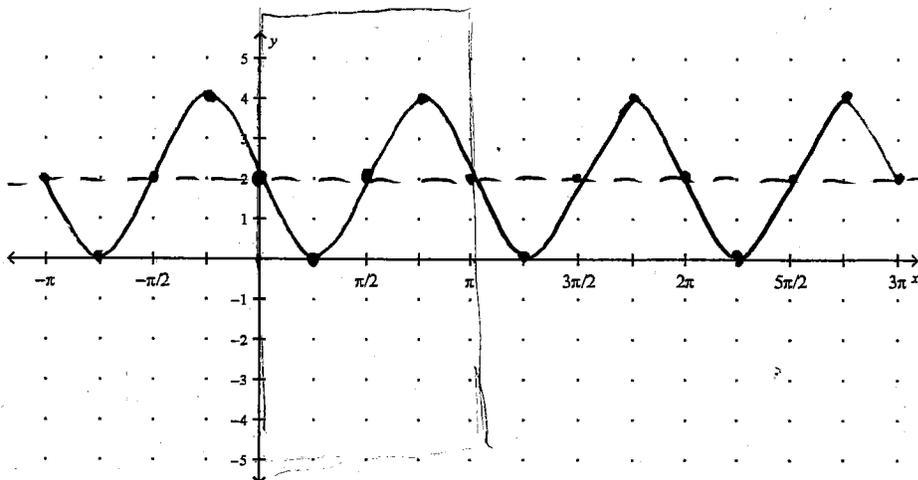
vertex:

$$(3, -1)$$

A.O.S. ↓

$$x = 3$$

13. Graph the equation $y = -2\sin 2x + 2$.



Orientation: neg

Amp: 2

Period: $\frac{2\pi}{2} = \pi$
length: 2

vertical shift: 2

Phase shift: None

Max: 4

min: 0

14. Convert each of the angles to radian measure.

a. $40^\circ \cdot \frac{\pi}{180} = \frac{40}{180} \pi$ b. $170^\circ \cdot \frac{\pi}{180} = \frac{170}{180} \pi$ c. $300^\circ \cdot \frac{\pi}{180} = \frac{300}{180} \pi$

$\frac{2\pi}{9}$ radians

$\frac{17\pi}{18}$ radians

$= \frac{5\pi}{3}$ radians

15. Convert each of the angles to degree measure.

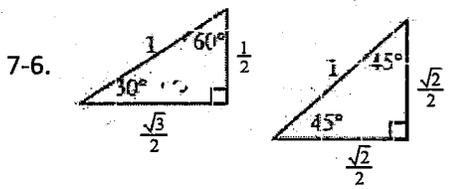
a. $\frac{9\pi}{4} \cdot \frac{180}{\pi} = \frac{9 \cdot 180}{4}$ b. $\frac{2\pi}{5} \cdot \frac{180}{\pi} = \frac{2 \cdot 180}{5}$ c. $\frac{13\pi}{10} \cdot \frac{180}{\pi} = \frac{13 \cdot 180}{10}$

$= 405^\circ$

$= 72^\circ$

$= 234^\circ$

Solutions to Chapter 7 Final Exam Study Guide



7-54. $P(\cos 50, \sin 50) \rightarrow P(0.64, 0.77)$,
 $Q(\cos 110, \sin 110) \rightarrow Q(-0.34, 0.94)$

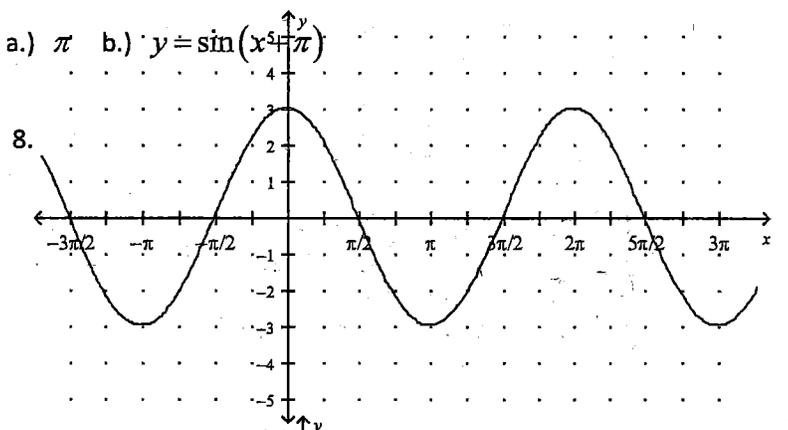
7-62. $58^\circ, 122^\circ, 238^\circ, 302^\circ$

7-53. $(\frac{\sqrt{13}}{4}, \frac{1}{4})$

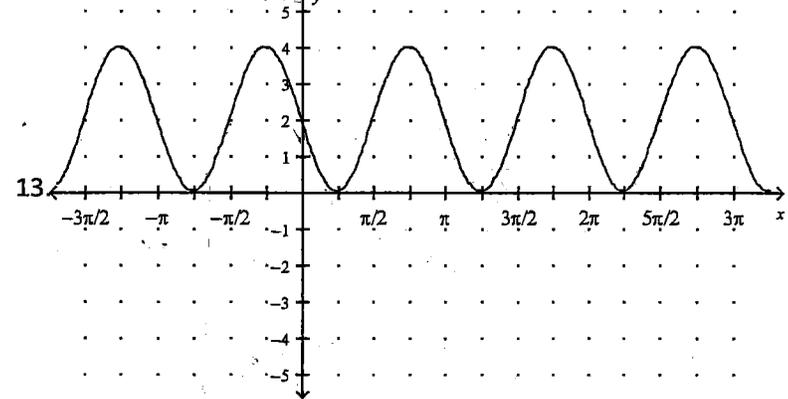
7-110. a.) $-\frac{5}{13}$ b.) $\frac{5}{12}$

7-117. a.) π b.) $y = \sin(x - \frac{\pi}{4})$

7-129. a.) $y = \sin(x - \frac{\pi}{4}) + 2$
 b.) $y = 1.5 \sin(x + \frac{\pi}{2}) + 0.5$
 or $y = 1.5 \cos(x) + 0.5$



7-40. x: $(\frac{-3+\sqrt{6}}{3}, 0)$ $(\frac{-3-\sqrt{6}}{3}, 0)$ y: $(0, 1)$



10. $y = (x+2)^2 - 21$

V: $(-2, -21)$, Y-int: $(0, -17)$

11. $y = 3(x+1)^2 - 2$
 $(-0.2, 0)$ $(-1.9, 0)$

7-82. $y = 3(x-3)^2 - 1$
 V: $(3, -1)$ a.o.s. $x = 3$

14. a. $\frac{2\pi}{9}$ b. $\frac{17\pi}{18}$ c. $\frac{5\pi}{3}$

15. a. 405° b. 72° c. 234°

1) Without using a calculator, answer the following questions about

$$f(x) = -2x^3(x-3)(x+5)(x-4)^2$$

$$3 + 1 + 1 + 2$$

a) What is the degree of the polynomial?

$$3 + 1 + 1 + 2 = 7^{\text{th}} \text{ degree}$$

b) Is the orientation positive or negative?

$$-2x^3 \text{ negative orientation}$$

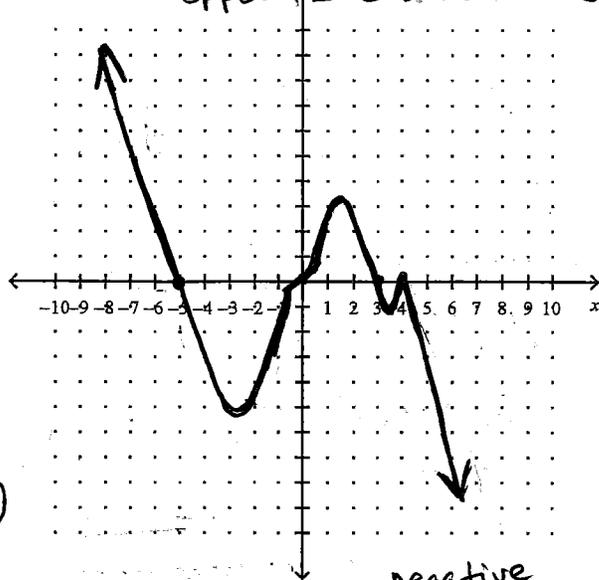
c) How many different roots are there?

$$0, 3, -5, 4 \text{ Four different roots}$$

d) What are the roots?

$$0 \text{ (triple root)}, 3, -5, 4 \text{ (double root)}$$

e) Sketch a graph



7th degree
Odd functions have opposite end behaviors.

negative orientation goes down to the right!!

2) Write the equation for the given situations.

a) A polynomial with roots at -4, -3, and a double root at 2 that also passes through (3, -14).

* Turn roots into factors.
Then create equation for function. Include "a" to find by plugging in a non-root point for x & y and solving for "a".

$$y = a(x+4)(x+3)(x-2)^2$$

root:	factor
-4	(x+4)
-3	(x+3)
2 (dr)	(x-2) ²

$$-14 = a(3+4)(3+3)(3-2)^2$$

$$-14 = a(7)(6)(1)$$

$$\frac{-14}{42} = \frac{42a}{42}$$

$$-\frac{1}{3} = a$$

$$y = -\frac{1}{3}(x+4)(x+3)(x-2)^2$$

b) A polynomial with roots at 0, 1, 4, and -5 that also passes through (5, 800).

root:	factor:	
0	x	$y = ax(x-1)(x-4)(x+5)$
1	(x-1)	$800 = a(5)(5-1)(5-4)(5+5)$
4	(x-4)	$800 = a(5)(4)(1)(10)$
-5	(x+5)	$\frac{800}{200} = \frac{200a}{200}$

$$y = 4x(x-1)(x-4)(x+5)$$

$$4 = a$$

3) Simplify:

(Simplify)

* Remember that even exponents
divisible by 4 are equal to "1".
Even exponents not divisible by 4
equal "-1".

a) i^{31}
 $i^{30} \cdot i$
 $1 \cdot i = -i$

b) $(3-2i)^2 = (3-2i)(3-2i)$
 $9 - 6i - 6i + 4i^2$
 $9 - 6i - 6i + 4(-1)$
 $9 - 6i - 6i - 4$
 $5 - 12i$

c) $(6-5i)(2+3i)$
 $12 + 18i - 10i - 15i^2$
 $12 + 18i - 10i - 15(-1)$
 $12 + 18i - 10i + 15$
 $27 + 8i$

d) $(2i)^2(5i)$
 $4i^2(5i)$
 $4(-1)(5i)$
 $-4(5i)$
 $-20i$

e) $(2+7i) + (-3+5i) *$
 $2 + (-3) = -1$
 $7i + 5i = 12i$
 $-1 + 12i$

* This is adding!!
Not multiplying!!

4) Write a possible equation in standard form that has the given roots.

a) 3, 2i, -2i b) 3-2i, 3+2i

Roots: Factors:

3 x-3

2i x-2i

-2i x+2i

$= (x-2i)(x+2i)(x-3)$

$= (x^2 - 4i^2)(x-3)$

$= (x^2 + 4)(x-3)$

$+4 \begin{array}{|c|c|c|} \hline 4x & -12 & \\ \hline x^2 & x^3 & -3x \\ \hline \end{array} = y = x^3 - 3x^2 + 4x - 12$

roots: factors:

3-2i (x-3+2i)

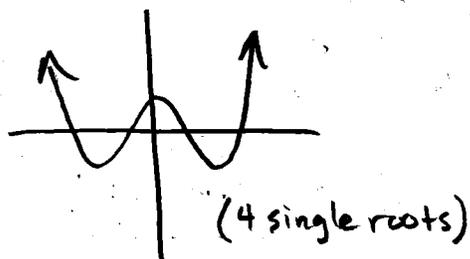
3+2i (x-3-2i)

$+2i \begin{array}{|c|c|c|} \hline 2xi & -6i & -4i^2 \\ \hline -3 & -3x & 9 \\ \hline x & x^2 & -3x \\ \hline \end{array} = y = x^2 - 6x + 13$

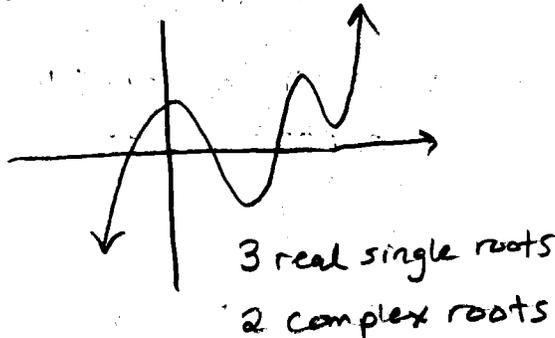
x -3 -2i

5) Sketch a graph for the following situations.

a) A polynomial with 4 real roots



b) A polynomial with 3 real 2 complex roots



6) Solve: $3x^2 + 5x + 4 = 0$

Does not touch x-axis, so you
must use quadratic formula!

$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$ a=3 b=5 c=4

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(4)}}{2(3)} = \frac{-5 \pm \sqrt{25 - 48}}{6} = \frac{-5 \pm \sqrt{-23}}{6}$

$x = \frac{-5 \pm i\sqrt{23}}{6}$

(Remember... complex
roots always come
in pairs!)

7) Divide the following.

Remember to use root if using synthetic division.

a) $(x^3 - 3x^2 + 3x - 2) \div (x - 2)$

	x^2	$-x$	1
x	x^3	$-x^2$	x
-2	$-2x^2$	$2x$	-2

$x^3 - 3x^2 + 3x - 2$

$x^2 - x + 1$

OR

2	1	-3	3	-2
		2	-2	2
	1	-1	1	0

b) $\frac{6x^4 - 7x^3 + 11x^2 - 9x + 2}{3x - 2}$

	$2x^3$	$-x^2$	$3x$	-1
$3x$	$6x^4$	$-3x^3$	$9x^2$	$-3x$
-2	$-4x^3$	$2x^2$	$-6x$	2

$6x^4 - 7x^3 + 11x^2 - 9x + 2$

$2x^3 - x^2 + 3x - 1$

c) $(6x^3 + 11x^2 - 12x - 1) \div (3x + 1)$

	$2x^2$	$3x$	-5
$3x$	$6x^3$	$9x^2$	$-15x$
$+1$	$2x^2$	$3x$	-5

$6x^3 + 11x^2 - 12x - 1$

$2x^2 + 3x - 5 + \frac{4}{3x+1}$

d) $\frac{x^5 - 1}{x - 1}$

* FILL IN MISSING DEGREES WITH "0's"

	x^4	x^3	x^2	x	1
x	x^5	x^4	x^3	x^2	x
-1	$-x^4$	$-x^3$	$-x^2$	$-x$	-1

$x^5 + 0x^4 + 0x^3 + 0x^2 + 0x - 1$

$x^4 + x^3 + x^2 + x + 1$

8) Factor the following polynomials completely and find all the roots.

a) $y = x^4 + 2x^3 + 10x^2 + 18x + 9$

* Remember to graph on calculator to find integral roots!

Possible integral roots: $\pm 1, \pm 3, \pm 9$

Integral roots: -1 (double root)

Known factors: $(x+1)(x+1)$

-1	1	2	10	18	9
		-1	-1	-9	-9
	1	1	9	9	0

-1	1	1	9	9
		-1	0	-9
	1	0	9	0

$= (x+1)(x^3 + x^2 + 9x + 9)$

$= (x+1)(x+1)(x^2 + 9)$

$0 = (x+1)(x+1)(x^2 + 9)$

$x+1=0 \quad x+1=0 \quad x^2+9=0$

$x=-1 \quad x=-1 \quad x^2=-9$

Roots: $\{-1(\text{dr}), \pm 3i\}$

$\sqrt{x^2} = \sqrt{-9}$

$|x| = 3i$

$x = \pm 3i$

b) $f(x) = x^5 - 4x^3 - x^2 + 4$

Possible integral roots: $\pm 1, \pm 2, \pm 4$

Integral roots: $-2, 1, 2$

Known factors: $(x+2)(x-1)(x-2)$

$0 = (x+2)(x-1)(x-2)(x^2 + x + 1)$

$x+2=0 \quad x-1=0 \quad x-2=0 \quad x^2+x+1=0$

$x=-2 \quad x=1 \quad x=2$

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$\frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

-2	1	0	-4	-1	0	4
		-2	4	0	2	-4
	1	-2	0	-1	2	0

1	1	-2	0	-1	2
		1	-1	-1	-2
	1	-1	-1	-2	0

2	1	-1	-1	-2
		2	2	2
	1	1	1	0

$= (x+2)(x^4 - 2x^3 + 0x^2 - x + 2)$

$= (x+2)(x-1)(x^3 - x^2 - x - 2)$

$= (x+2)(x-1)(x-2)(x^2 + x + 1)$

Roots: $\{-2, 1, 2, \frac{-1 \pm i\sqrt{3}}{2}\}$

Algebra II
Chapter 8 Exam Review

Answers:

1. a) 7

b) negative

c) 4

d) 0, 3, 4, -5

e) graph

3. a) $-i$

b) $5-12i$

c) $27+8i$

d) $-20i$

e) $-1+12i$

8. a) $(x+1)^2(x^2+9); x=-1, 3i, -3i$

b) $(x+2)(x-1)(x-2)(x^2+x+1); x=-2, 1, 2, \frac{-1 \pm i\sqrt{3}}{2}$

2. a) $-\frac{1}{3}(x+4)(x+3)(x-2)^2$

b) $4x(x-1)(x-4)(x+5)$

4. a) $x^3-3x^2+4x-12$

b) $x^2-6x+13$

5. Varies

6. $x = \frac{-5 \pm i\sqrt{23}}{6}$

7. a) x^2-x+1

b) $2x^3-x^2+3x-1$

c) $2x^2+3x-5+\frac{4}{3x+1}$

d) $x^4+x^3+x^2+x+1$

Chapter 10 Exam Review
Algebra 2

Name _____

Determine if the sequence is arithmetic or geometric. Write an expression for the n^{th} of the sequence.

1. $3, 6, 9, 12, \dots$ Arithmetic ($d=3$)

$$a_n = 3 + 3(n-1)$$

2. $21, 16, 11, 6, \dots$ Arithmetic ($d=-5$)

$$a_n = 21 - 5(n-1)$$

3. $1, 2, 4, 7, 11, 16, \dots$

Neither

4. $4, 12, 36, 108, \dots$ Geometric ($r=3$)

$$a_n = 4(3)^{n-1}$$

5.) Consider the series $21 + 17 + 13 + \dots + -99$. Write an expression for the n^{th} term.

$$a_n = 21 - 4(n-1) \quad \leftarrow \text{nth term expression}$$

a.) How many terms are in the series?

$$\begin{array}{r} -99 = 21 - 4(n-1) \\ -21 \quad -21 \\ \hline -120 = -4(n-1) \end{array}$$

$$\begin{array}{r} -120 = -4(n-1) \\ \frac{-120}{-4} = \frac{-4(n-1)}{-4} \\ 30 = n-1 \\ 31 = n \end{array}$$

There are 31 terms

b.) Find the sum of the series.

$$a_1 = 21$$

$$a_{31} = -99$$

$$n = 31$$

$$S_{31} = \frac{31}{2}(21 + (-99)) = -1209$$

$$S_{31} = -1209$$

6.) Find the sum of each series.

a.) $7 + 10 + 13 + \dots + 73$

$$a_n = 7 + 3(n-1) \quad S_{23} = \frac{23}{2}(7+73)$$

$$\begin{array}{r} 73 = 7 + 3(n-1) \\ -7 \quad -7 \\ \hline 66 = 3(n-1) \end{array}$$

$$S_{23} = 920$$

$$\frac{66}{3} = \frac{3(n-1)}{3}$$

$$22 = n-1$$

$$23 = n$$

b.) $\sum_{n=1}^{15} (10+5n) \quad n = (15-1)+1 = 15$

$$a_1 = 10 + 5(1) = 15$$

$$a_{15} = 10 + 5(15) = 85$$

$$S_{15} = \frac{15}{2}(15+85) = 750$$

$$S_{15} = 750$$

7.) Consider the series $3125 + 625 + 125 \dots + 0.04$. Write an expression for the n^{th} term.

$$a_n = 3125 \left(\frac{1}{5}\right)^{n-1}$$

$$r = \frac{625}{3125} = \frac{1}{5}$$

a.) How many terms are in the series?

$$\frac{0.04}{3125} = \frac{3125 \left(\frac{1}{5}\right)^{n-1}}{3125}$$

$$0.0000128 = \left(\frac{1}{5}\right)^{n-1}$$

There are 8 terms in series.

b.) Find the sum of the series.

$$S_8 = \frac{3125 \left(1 - \left(\frac{1}{5}\right)^8\right)}{1 - \left(\frac{1}{5}\right)}$$

$$\log(0.0000128) = \log\left(\frac{1}{5}\right)^{n-1}$$

$$\frac{\log(0.0000128)}{\log\left(\frac{1}{5}\right)} = \frac{(n-1) \cdot \log\left(\frac{1}{5}\right)}{\log\left(\frac{1}{5}\right)}$$

$$S_8 = 3906.24$$

$$\begin{aligned} 7 &= n-1 \\ 8 &= n \end{aligned}$$

8.) Find the sum of each series.

a.) $2916 + 972 + 324 + \dots + 4$ $r = \frac{972}{2916} = \frac{1}{3}$

$$a_n = 2916 \left(\frac{1}{3}\right)^{n-1}$$

$$4 = 2916 \left(\frac{1}{3}\right)^{n-1}$$

$$\frac{1}{729} = \left(\frac{1}{3}\right)^{n-1}$$

$$\frac{\log\left(\frac{1}{729}\right)}{\log\left(\frac{1}{3}\right)} = n-1$$

$$\begin{aligned} 6 &= n-1 \\ 7 &= n \end{aligned}$$

$$S_7 = \frac{2916 \left(1 - \left(\frac{1}{3}\right)^7\right)}{1 - \frac{1}{3}}$$

$$S_7 = 4372$$

b.) $\sum_{n=1}^{12} 5120 \left(\frac{1}{2}\right)^{n-1}$

$$n = (12-1) + 1 = 12$$

$$a_1 = 5120 \left(\frac{1}{2}\right)^{1-1} = 5120$$

$$a_{12} = 5120 \left(\frac{1}{2}\right)^{12-1} = 2.5$$

$$S_{12} = \frac{5120 \left(1 - \left(\frac{1}{2}\right)^{12}\right)}{1 - \left(\frac{1}{2}\right)}$$

$$S_{12} = 10,237.5$$

9.) For each infinite geometric series, find the sum.

$$S_{\infty} = \frac{a_1}{1-r}$$

a.) $972 + 324 + 108, \dots$

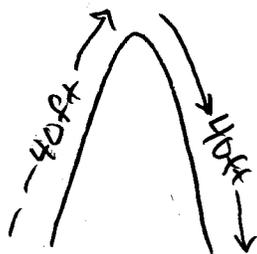
$$r = \frac{324}{972} = \frac{1}{3}$$

$$S_{\infty} = \frac{972}{1 - \frac{1}{3}} = 1458$$

b.) $\sum_{n=1}^{\infty} 600 \left(\frac{1}{4}\right)^{n-1}$

$$S_{\infty} = \frac{600}{1 - \frac{1}{4}} = \frac{600}{\frac{3}{4}} = 800$$

10.) While sitting on the gymnasium floor, Levi through a basketball 40 ft. into the air. The rebound ratio of the basketball is 0.7. If the ball is permitted to bounce forever, what is the total distance the ball will travel?



1st bounce travels 80ft.

$$S_{\infty} = \frac{80}{1-0.7} = \frac{80}{0.3} = 266.\bar{6} \text{ ft}$$

Solutions:

1.) $t(n) = 3 + 3(n-1)$ or $t(n) = 3n$

2. $t(n) = 21 - 5(n-1)$ or $t(n) = 26 - 5n$

3.) neither

4.) $t(n) = 4(3)^{n-1}$ or $t(n) = \frac{4}{3}(3)^n$

5.) $t(n) = 21 - 4(n-1)$ or $t(n) = 25 - 4n$; 31; -1209

6.) 920; 750

7.) $t(n) = 3125\left(\frac{1}{5}\right)^{n-1}$ or $t(n) = 15625\left(\frac{1}{5}\right)^n$; 8; 3906.24

8.) 4372; 10237.5

9.) 1458; 800

10.) $266.\bar{6}$ ft

Decide if each of the following statements are always true, sometimes true, or never true. If a statement is always true, justify how you know; if it is sometimes true, give the exact values of x that make it true; and if it is never true, explain why it is never true.

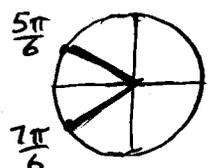
1) $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$ Always true.
Graph $y = \sin(x)$ & $y = \cos\left(\frac{\pi}{2} - x\right)$ on calculator (radians) or look at the table. Same graphs.

2) $\tan \theta = 1$ Sometimes true.
 $\tan \theta = 1$ at $\frac{\pi}{4}$ & $\frac{5\pi}{4}$ (from 0 to 2π)
or $\frac{\pi}{4} + \pi n$ for all solutions

For each of the following equations, find the solutions that lie in the domain $0 \leq x < 2\pi$.

3) $\frac{2\sin \theta}{2} = \frac{\sqrt{2}}{2}$
 $\sin \theta = \frac{\sqrt{2}}{2}$

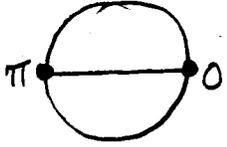
 $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$
Quad I & II
Ref $\angle = \frac{\pi}{4}$

4) $\frac{2\cos \theta}{2} = -\frac{\sqrt{3}}{2}$
 $\cos \theta = -\frac{\sqrt{3}}{2}$

 $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$
Quad II & III
Ref $\angle = \frac{\pi}{6}$

For each of the following equations, find all solutions.

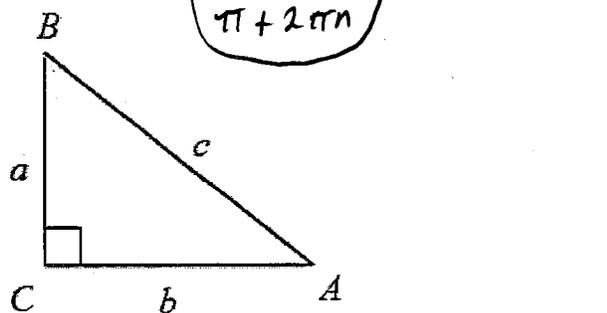
5) $4\sin \theta + 2 = 0$
 $\frac{4\sin \theta}{4} = \frac{-2}{4}$
 $\sin \theta = -\frac{1}{2}$

 $\theta = \frac{7\pi}{6} + 2\pi n$
 $\theta = \frac{11\pi}{6} + 2\pi n$

6) $4\cos^2 \theta - 4 = 0$
 $4\cos^2 \theta = 4$
 $\sqrt{\cos^2 \theta} = \sqrt{1}$
 $\cos \theta = \pm 1$
 $\cos \theta = 1$ $\cos \theta = -1$
 $0 + 2\pi n$ OR $0 + 2\pi n$
 $\pi + 2\pi n$


7) Using the triangle below, find:

a. $\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{a}{c}$
b. $\sec A = \frac{\text{hyp}}{\text{adj}} = \frac{c}{b}$
c. $\csc B = \frac{\text{hyp}}{\text{opp}} = \frac{c}{b}$
d. $\tan B = \frac{\text{opp}}{\text{adj}} = \frac{b}{a}$



For each of the following equations, find the solutions that lie in the domain $0 \leq x < 2\pi$.

8) $\sec^2 \theta + 3 = 4$
 $\sec^2 \theta = 1$
 $\sqrt{\sec^2 \theta} = \sqrt{1}$
 $|\sec \theta| = 1$
 $\sec \theta = \pm 1$
 $\sec \theta = \frac{1}{\cos \theta}$
 $\cos \theta = \frac{1}{1} = 1$ $\cos \theta = \frac{1}{-1} = -1$
 $\theta = 0$ $\theta = \pi$

9) $\cot \theta = \sqrt{3}$
 $\tan \theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
Remember: $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
 $\tan \theta = \frac{\sqrt{3}}{3}$
 $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$
Ref $\angle = \frac{\pi}{6}$
Quad: I & III


Ignore the " $+2\pi k$ "
★ on the answer space on the back. (for 8 & 9)

10) Rewrite each of the following expressions in terms of either $\sin(\theta)$ or $\cos(\theta)$.

a) $\tan(\theta) = \frac{\sin \theta}{\cos \theta}$

b) $\csc(\theta) = \frac{1}{\sin \theta}$

c) $\cot(\theta) = \frac{\cos \theta}{\sin \theta}$

d) $\sec(\theta) = \frac{1}{\cos \theta}$

Verify each identity.

11) $\tan \theta \sin \theta \cos \theta \csc^2 \theta = 1$

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{1} \cdot \frac{\cos \theta}{1} \cdot \frac{1}{\sin^2 \theta}$$

$$\frac{\sin^2 \theta \cdot \cos \theta}{\cos \theta \cdot \sin^2 \theta}$$

$$\boxed{1 = 1}$$

12) $\sin^2 \theta + \sin^2 \theta \tan^2 \theta = \tan^2 \theta$

$$\sin^2 \theta [1 + \tan^2 \theta]$$

$$\sin^2 \theta [\sec^2 \theta]$$

$$\frac{\sin^2 \theta}{1} \cdot \frac{1}{\cos^2 \theta}$$

$$\boxed{\frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta}$$

13) $\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta} = \tan^2 \theta$

$$\frac{\sec^2 \theta}{\csc^2 \theta} = \frac{\sec^2 \theta}{1} \cdot \frac{1}{\csc^2 \theta}$$

$$= \frac{1}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{1}$$

$$= \boxed{\frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta}$$

14) $\sin\left(\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{\pi}{3}\right) = \cos \theta$

(OMIT)

ANSWERS

1) Always true

2) Sometimes true, $\theta = \frac{\pi}{4} + 2\pi k, \frac{5\pi}{4} + 2\pi k$

3) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

4) $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

5) $\theta = \frac{7\pi}{6} + 2\pi k, \frac{11\pi}{6} + 2\pi k$

6) $\theta = 0 + 2\pi k, \pi + 2\pi k$

7a) $\frac{a}{c}$

7b) $\frac{c}{b}$

7c) $\frac{c}{b}$

7d) $\frac{b}{a}$

8) $\theta = 0, \pi$

9) $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$

10a) $\frac{\sin \theta}{\cos \theta}$

10b) $\frac{1}{\sin \theta}$

10c) $\frac{\cos \theta}{\sin \theta}$

10d) $\frac{1}{\cos \theta}$

11-14) Proofs ©